Conversions Between S, Z, Y, h, $ABCD$, and T Parameters which are Valid for Complex Source and Load Impedances

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***Abstract*— This paper provides tables which contain the conversion between the various common two-port parameters, *Z*, *Y*, *h*, *ABCD*, *S*, and *T*. The conversion are valid for complex normalizing impedances. An example is provided which verifies the conversions to and from S parameters.**

I. INTRODUCTION

MOST microwave textbooks these days seem to provide a table of the conversion between the various 2-port parameters. These 2-port parameters often include Z (impedance), Y (admittance), h (hybrid), $ABCD$ (chain), S (scattering), and T (chain scattering or chain transfer). While the scattering parameters have been shown [1] to be valid for complex normalizing impedances (with positive real parts), the tables in [2]–[15] are not valid for complex source and load impedances. Often, the tables only provide conversions for the cases where port 1 and port 2 normalizing impedances are equal, i.e., $Z_{01} = Z_{02} = Z_0$. Some have results in which Z_{01} and Z_{02} are normalized to 1. Others provide equations for port 1 and port 2 impedances Z_{01} and Z_{02} to be unique. However, in all of these cases, the results are not valid when the impedances, Z_{01} and Z_{02}, or just Z_0, are complex.

Of the two-port parameters mentioned, only the S and T parameters are dependent upon the source and load impedances. In this paper, the derivations of the conversions from the S and T parameters to the other 2-port parameters includes complex source and load impedances. The equations developed in this work are valid with port 1 and port 2 normalizing impedances complex and unique. When the normalizing impedances are real, the results simplify to those shown in other references. To make the list complete, the conversions between the Z, Y, h, and $ABCD$ parameters as well as between S and T parameters are included.

II. DERIVATION

Two-port parameters are defined for a general 2-port network as shown in Fig. 1. Using the voltages and currents defined in this figure, the various 2-port parameters are written as

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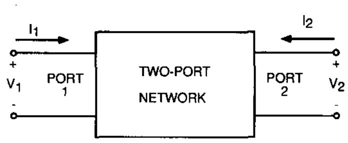

Fig. 1. A general two-port network with voltages and currents defined.

Z parameters

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2 \tag{1a}$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2, \tag{1b}$$

Y parameters

$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2 \tag{2a}$$

$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2, \tag{2b}$$

h parameters

$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2 \tag{3a}$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2, \tag{3b}$$

$ABCD$ parameters

$$V_1 = A \cdot V_2 - B \cdot I_2 \tag{4a}$$

$$I_1 = C \cdot V_2 - D \cdot I_2, \tag{4b}$$

S parameters

$$b_1 = S_{11} \cdot a_1 + S_{12} \cdot a_2 \tag{5a}$$

$$b_2 = S_{21} \cdot a_1 + S_{22} \cdot a_2, \tag{5b}$$

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TABLE I

EQUATIONS FOR THE CONVERSION BETWEEN S PARAMETERS AND Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$S_{11} = \frac{(Z_{11}-Z_{01}^*)(Z_{22}+Z_{02})-Z_{12}Z_{21}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{11} = \frac{(Z_{01}^*+S_{11}Z_{01})(1-S_{22})+S_{12}S_{21}Z_{01}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{12} = \frac{2Z_{12}(R_{01}R_{02})^{1/2}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{12} = \frac{2S_{12}(R_{01}R_{02})^{1/2}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{21} = \frac{2Z_{21}(R_{01}R_{02})^{1/2}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{21} = \frac{2S_{21}(R_{01}R_{02})^{1/2}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{22} = \frac{(Z_{11}+Z_{01})(Z_{22}-Z_{02}^*)-Z_{12}Z_{21}}{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}$	$Z_{22} = \frac{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$
$S_{11} = \frac{(1-Y_{11}Z_{01}^*)(1+Y_{22}Z_{02})+Y_{12}Y_{21}Z_{01}^*Z_{02}}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{11} = \frac{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{12} = \frac{-2Y_{12}(R_{01}R_{02})^{1/2}}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{12} = \frac{-2S_{12}(R_{01}R_{02})^{1/2}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{21} = \frac{-2Y_{21}(R_{01}R_{02})^{1/2}}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{21} = \frac{-2S_{21}(R_{01}R_{02})^{1/2}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{22} = \frac{(1+Y_{11}Z_{01})(1-Y_{22}Z_{02}^*)+Y_{12}Y_{21}Z_{01}Z_{02}^*}{(1+Y_{11}Z_{01})(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}Z_{02}}$	$Y_{22} = \frac{(Z_{01}^*+S_{11}Z_{01})(1-S_{22})+S_{12}S_{21}Z_{01}}{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}$
$S_{11} = \frac{(h_{11}-Z_{01}^*)(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{11} = \frac{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{12} = \frac{2h_{12}(R_{01}R_{02})^{1/2}}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{12} = \frac{2S_{12}(R_{01}R_{02})^{1/2}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{21} = \frac{-2h_{21}(R_{01}R_{02})^{1/2}}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{21} = \frac{-2S_{21}(R_{01}R_{02})^{1/2}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{22} = \frac{(Z_{01}+h_{11})(1-h_{22}Z_{02}^*)+h_{12}h_{21}Z_{02}^*}{(Z_{01}+h_{11})(1+h_{22}Z_{02})-h_{12}h_{21}Z_{02}}$	$h_{22} = \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}$
$S_{11} = \frac{AZ_{02}+B-CZ_{01}^*Z_{02}-DZ_{01}^*}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$A = \frac{(Z_{01}^*+S_{11}Z_{01})(1-S_{22})+S_{12}S_{21}Z_{01}}{2S_{21}(R_{01}R_{02})^{1/2}}$
$S_{12} = \frac{2(AD-BC)(R_{01}R_{02})^{1/2}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$B = \frac{(Z_{01}^*+S_{11}Z_{01})(Z_{02}^*+S_{22}Z_{02})-S_{12}S_{21}Z_{01}Z_{02}}{2S_{21}(R_{01}R_{02})^{1/2}}$
$S_{21} = \frac{2(R_{01}R_{02})^{1/2}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$C = \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}(R_{01}R_{02})^{1/2}}$
$S_{22} = \frac{-AZ_{02}^*+B-CZ_{01}Z_{02}^*+DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$	$D = \frac{(1-S_{11})(Z_{02}^*+S_{22}Z_{02})+S_{12}S_{21}Z_{02}}{2S_{21}(R_{01}R_{02})^{1/2}}$

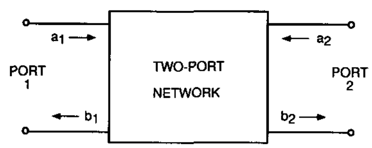

Fig. 2. A general two port network with a's and b's defined.

T parameters[1]

$$a_1 = T_{11} \cdot b_2 + T_{12} \cdot a_2 \qquad (6a)$$

$$b_1 = T_{21} \cdot b_2 + T_{22} \cdot a_2 \qquad (6b)$$

where the a's and b's are shown in Fig. 2 and defined below.

$$a_j = \left[\frac{Z_{0j} + Z_{0j}^*}{2}\right]^{1/2} \cdot I_{ji} \qquad (7a)$$

$$b_j = \left[\frac{Z_{0j} + Z_{0j}^*}{2}\right]^{1/2} \cdot I_{jr} \qquad (7b)$$

[1]Some authors, (e.g. Rizzi [16]) define the T parameters as $b_1 = T_{11} \cdot a_2 + T_{12} \cdot b_2$, and $a_1 = T_{21} \cdot a_2 + T_{22} b_2$. In this case, the parameters can just be switched from what is derived in this paper. T_{11} and T_{22} are switched, T_{12} and T_{21} are switched.

where * indicates complex conjugate and Z_{0j} is the normalizing impedance for the jth port. For two-port networks, Z_{01} and Z_{02} are the source and load impedances of the system in which the S parameters of the two-port are measured or calculated. I_{ji} and I_{jr} are the incident and reflected currents for the jth port. Knowing that,

$$I_j = I_{ji} - I_{jr} \qquad (8)$$

we can solve (7a) and (7b) for I_{ji} and I_{jr} and substitute them into (8) to get,

$$I_j = \left[\frac{2}{Z_{0j} + Z_{0j}^*}\right]^{1/2} \cdot (a_j - b_j). \qquad (9)$$

Knowing also that,

$$V_j = V_{ji} + V_{jr} \qquad (10)$$

where V_{ji} and V_{jr} are the incident and reflected voltage at the jth port, we can substitute the expressions for I_{ji} and I_{jr} along with

$$V_{ji} = I_{ji} \cdot Z_{0j}^* \qquad V_{jr} = I_{jr} \cdot Z_{0j}$$

into (10) to get,

$$V_j = \left[\frac{2}{Z_{0j} + Z_{0j}^*}\right]^{1/2} \cdot (a_j \cdot Z_{0j}^* + b_j \cdot Z_{0j}). \qquad (11)$$

TABLE II

EQUATIONS FOR THE CONVERSION BETWEEN T PARAMETERS AND Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$T_{11} = \frac{(Z_{11}+Z_{01})(Z_{22}+Z_{02})-Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{11} = \frac{Z_{01}^*(T_{11}+T_{12})+Z_{01}(T_{21}+T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{12} = \frac{(Z_{11}+Z_{01})(Z_{02}^*-Z_{22})+Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{12} = \frac{2(R_{01}R_{02})^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{21} = \frac{(Z_{11}-Z_{01}^*)(Z_{22}+Z_{02})-Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{21} = \frac{2(R_{01}R_{02})^{1/2}}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{22} = \frac{(Z_{01}^*-Z_{11})(Z_{22}-Z_{02}^*)+Z_{12}Z_{21}}{2Z_{21}(R_{01}R_{02})^{1/2}}$	$Z_{22} = \frac{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}-T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$
$T_{11} = \frac{(-1-Y_{11}Z_{01})(1+Y_{22}Z_{02})+Y_{12}Y_{21}Z_{01}Z_{02}}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{11} = \frac{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}-T_{22})}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{12} = \frac{(1+Y_{11}Z_{01})(1-Y_{22}Z_{02}^*)+Y_{12}Y_{21}Z_{01}Z_{02}^*}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{12} = \frac{-2(R_{01}R_{02})^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{21} = \frac{(Y_{11}Z_{01}^*-1)(1+Y_{22}Z_{02})-Y_{12}Y_{21}Z_{01}^*Z_{02}}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{21} = \frac{-2(R_{01}R_{02})^{1/2}}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{22} = \frac{(1-Y_{11}Z_{01}^*)(1-Y_{22}Z_{02}^*)-Y_{12}Y_{21}Z_{01}^*Z_{02}^*}{2Y_{21}(R_{01}R_{02})^{1/2}}$	$Y_{22} = \frac{Z_{01}^*(T_{11}+T_{12})+Z_{01}(T_{21}+T_{22})}{T_{11}Z_{01}^*Z_{02}^*-T_{12}Z_{01}^*Z_{02}+T_{21}Z_{01}Z_{02}^*-T_{22}Z_{01}Z_{02}}$
$T_{11} = \frac{(-h_{11}-Z_{01})(1+h_{22}Z_{02})+h_{12}h_{21}Z_{02}}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{11} = \frac{Z_{02}^*(T_{11}Z_{01}^*+T_{21}Z_{01})-Z_{02}(T_{12}Z_{01}^*+T_{22}Z_{01})}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{12} = \frac{(h_{11}+Z_{01})(1-h_{22}Z_{02}^*)+h_{12}h_{21}Z_{02}^*}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{12} = \frac{2(R_{01}R_{02})^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{21} = \frac{(Z_{01}^*-h_{11})(1+h_{22}Z_{02})+h_{12}h_{21}Z_{02}}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{21} = \frac{-2(R_{01}R_{02})^{1/2}}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{22} = \frac{(h_{11}-Z_{01}^*)(1-h_{22}Z_{02}^*)+h_{12}h_{21}Z_{02}^*}{2h_{21}(R_{01}R_{02})^{1/2}}$	$h_{22} = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}+T_{22})}$
$T_{11} = \frac{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}{2(R_{01}R_{02})^{1/2}}$	$A = \frac{Z_{01}^*(T_{11}+T_{12})+Z_{01}(T_{21}+T_{22})}{2(R_{01}R_{02})^{1/2}}$
$T_{12} = \frac{AZ_{02}^*-B+CZ_{01}Z_{02}^*-DZ_{01}}{2(R_{01}R_{02})^{1/2}}$	$B = \frac{Z_{02}^*(T_{11}Z_{01}^*+T_{21}Z_{01})-Z_{02}(T_{12}Z_{01}^*+T_{22}Z_{01})}{2(R_{01}R_{02})^{1/2}}$
$T_{21} = \frac{AZ_{02}+B-CZ_{01}^*Z_{02}-DZ_{01}^*}{2(R_{01}R_{02})^{1/2}}$	$C = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{2(R_{01}R_{02})^{1/2}}$
$T_{22} = \frac{AZ_{02}^*-B-CZ_{01}^*Z_{02}^*+DZ_{01}^*}{2(R_{01}R_{02})^{1/2}}$	$D = \frac{Z_{02}^*(T_{11}-T_{21})-Z_{02}(T_{12}-T_{22})}{2(R_{01}R_{02})^{1/2}}$

Solving (9) and (11) for a_j and b_j gives

$$a_j = \frac{V_j + Z_{0j}I_j}{[2(Z_{0j} + Z_{0j}^*)]^{1/2}} \tag{12}$$

$$b_j = \frac{V_j - Z_{0j}^*I_j}{[2(Z_{0j} + Z_{0j}^*)]^{1/2}}. \tag{13}$$

Equations (12) and (13) are (3) and (4) in [1] and served as the starting point.

The notation, $S \leftrightarrow Z$, indicates the conversion from S parameters to Z parameters and Z parameters to S parameters. Since S and T parameters are defined in terms of a's and b's, they will contain the source and load normalizing impedances Z_{01} and Z_{02}. The other 2-port parameters are defined independent of the source and load impedances.

To derive the conversions, $S \leftrightarrow Z$, $S \leftrightarrow Y$, $S \leftrightarrow h$, $S \leftrightarrow ABCD$, $T \leftrightarrow Z$, $T \leftrightarrow Y$, $T \leftrightarrow h$, and $T \leftrightarrow ABCD$, it is necessary to use (9), (11)–(13). For example, to derive the expressions for S parameters in terms of the Z parameters, first substitute (9) and (11) into (1a) and (1b) and solve for b_1 and b_2 to get in the form of (5a) and (5b). Likewise, to get the expressions for the Z parameters in terms of the S parameters, substitute (12) and (13) into (5a) and (5b) and solve for V_1 and V_2 to get in the form of (1a) and (1b).

Since Z, Y, h, and $ABCD$ parameters do not require normalizing impedances, the conversions, $Z \leftrightarrow Y$, $Z \leftrightarrow h$, $Z \leftrightarrow ABCD$, $Y \leftrightarrow h$, $Y \leftrightarrow ABCD$, and $h \leftrightarrow ABCD$, as well as $S \leftrightarrow T$, are straight forward. These conversions are accomplished by rearranging one set of equations into the form of the other. These conversions appear in many of the references cited and are included here for completeness.

III. RESULTS

The results are given in the following tables. In these tables, Z_{01} and Z_{02} are the source and load impedances of the system to which the S and T parameters pertain. Complex conjugate is indicated by *, and R_{01} and R_{02} are the real parts of Z_{01} and Z_{02}.

Table I gives the conversions between S parameters and Z, Y, h, and $ABCD$ parameters. Table II gives the conversions

TABLE III
EQUATIONS FOR THE CONVERSION BETWEEN S PARAMETERS AND NORMALIZED Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$S_{11} = \frac{\left[Z_{11n} - \frac{Z_{01}^*}{Z_{01}}\right](Z_{22n}+1) - Z_{12n}Z_{21n}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$	$Z_{11n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right](1-S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$
$S_{12} = \frac{2Z_{12n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$	$Z_{12n} = \frac{2S_{12}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$
$S_{21} = \frac{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$	$Z_{21n} = \frac{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$
$S_{22} = \frac{(Z_{11n}+1)\left[Z_{22n} - \frac{Z_{02}^*}{Z_{02}}\right] - Z_{12n}Z_{21n}}{(Z_{11n}+1)(Z_{22n}+1) - Z_{12n}Z_{21n}}$	$Z_{22n} = \frac{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}$
$Z_{11n} = \frac{Z_{11}}{Z_{01}}$ $\quad Z_{12n} = \frac{Z_{12}}{(Z_{01}Z_{02})^{1/2}}$	$Z_{21n} = \frac{Z_{21}}{(Z_{01}Z_{02})^{1/2}}$ $\quad Z_{22n} = \frac{Z_{22}}{Z_{02}}$
$S_{11} = \frac{\left[1 - Y_{11n}\left[\frac{Z_{01}^*}{Z_{01}}\right]\right](1+Y_{22n}) + Y_{12n}Y_{21n}\left[\frac{Z_{01}^*}{Z_{01}}\right]}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$	$Y_{11n} = \frac{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$
$S_{12} = \frac{-2Y_{12n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$	$Y_{12n} = \frac{-2S_{12}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$
$S_{21} = \frac{-2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$	$Y_{21n} = \frac{-2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$
$S_{22} = \frac{(1+Y_{11n})\left[1 - Y_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right] + Y_{12n}Y_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{(1+Y_{11n})(1+Y_{22n}) - Y_{12n}Y_{21n}}$	$Y_{22n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right](1-S_{22}) + S_{12}S_{21}}{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}$
$Y_{11n} = Y_{11}Z_{01}$ $\quad Y_{12n} = Y_{12}(Z_{01}Z_{02})^{1/2}$	$Y_{21n} = Y_{21}(Z_{01}Z_{02})^{1/2}$ $\quad Y_{22n} = Y_{22}Z_{02}$
$S_{11} = \frac{\left[h_{11n} - \frac{Z_{01}^*}{Z_{01}}\right](1+h_{22n}) - h_{12n}h_{21n}}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$	$h_{11n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$
$S_{12} = \frac{2h_{12n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$	$h_{12n} = \frac{2S_{12}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$
$S_{21} = \frac{-2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$	$h_{21n} = \frac{-2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$
$S_{22} = \frac{(1+h_{11n})\left[1 - h_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right] + h_{12n}h_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{(1+h_{11n})(1+h_{22n}) - h_{12n}h_{21n}}$	$h_{22n} = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}$
$h_{11n} = \frac{h_{11}}{Z_{01}}$ $\quad h_{12n} = h_{12}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2}$	$h_{21n} = h_{21}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2}$ $\quad h_{22n} = h_{22}Z_{02}$
$S_{11} = \frac{A_n + B_n - C_n\left[\frac{Z_{01}^*}{Z_{01}}\right] - D_n\left[\frac{Z_{01}^*}{Z_{01}}\right]}{A_n + B_n + C_n + D_n}$	$A_n = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right](1-S_{22}) + S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$
$S_{12} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right](A_nD_n - B_nC_n)}{A_n + B_n + C_n + D_n} = \frac{2(AD-BC)}{A_n + B_n + C_n + D_n}$	$B_n = \frac{\left[\frac{Z_{01}^*}{Z_{01}} + S_{11}\right]\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] - S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$
$S_{21} = \frac{2}{A_n + B_n + C_n + D_n}$	$C_n = \frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$
$S_{22} = \frac{-A_n\left[\frac{Z_{02}^*}{Z_{02}}\right] + B_n - C_n\left[\frac{Z_{02}^*}{Z_{02}}\right] + D_n}{A_n + B_n + C_n + D_n}$	$D_n = \frac{(1-S_{11})\left[\frac{Z_{02}^*}{Z_{02}} + S_{22}\right] + S_{12}S_{21}}{2S_{21}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$
$A_n = \frac{AZ_{02}}{(R_{01}R_{02})^{1/2}}$ $\quad B_n = \frac{B}{(R_{01}R_{02})^{1/2}}$	$C_n = \frac{CZ_{01}Z_{02}}{(R_{01}R_{02})^{1/2}}$ $\quad D_n = \frac{DZ_{01}}{(R_{01}R_{02})^{1/2}}$

TABLE IV
EQUATIONS FOR THE CONVERSION BETWEEN T PARAMETERS AND NORMALIZED Z, Y, h, AND $ABCD$ PARAMETERS WITH A SOURCE IMPEDANCE Z_{01} AND LOAD IMPEDANCE Z_{02}

$$T_{11} = \frac{(Z_{11n}+1)(Z_{22n}+1)-Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Z_{11n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}\right](T_{11}+T_{12})+(T_{21}+T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$T_{12} = \frac{(Z_{11n}+1)\left[\frac{Z_{02}^*}{Z_{02}}-Z_{22n}\right]+Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Z_{12n} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$T_{21} = \frac{\left[Z_{11n}-\frac{Z_{01}^*}{Z_{01}}\right](Z_{22n}+1)-Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Z_{21n} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$T_{22} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}-Z_{11n}\right]\left[Z_{22n}-\frac{Z_{02}^*}{Z_{02}}\right]+Z_{12n}Z_{21n}}{2Z_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Z_{22n} = \frac{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}{T_{11}+T_{12}-T_{21}-T_{22}}$$

$$Z_{11n} = \frac{Z_{11}}{Z_{01}} \qquad Z_{12n} = \frac{Z_{12}}{(Z_{01}Z_{02})^{1/2}} \qquad Z_{21n} = \frac{Z_{21}}{(Z_{01}Z_{02})^{1/2}} \qquad Z_{22n} = \frac{Z_{22}}{Z_{02}}$$

$$T_{11} = \frac{(-1-Y_{11n})(1+Y_{22n})+Y_{12n}Y_{21n}}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Y_{11n} = \frac{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$T_{12} = \frac{(1+Y_{11n})\left[1-Y_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]+Y_{12n}Y_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Y_{12n} = \frac{-2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$T_{21} = \frac{\left[Y_{11n}\left[\frac{Z_{01}^*}{Z_{01}}\right]-1\right](1+Y_{22n})-Y_{12n}Y_{21n}\left[\frac{Z_{01}^*}{Z_{01}}\right]}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Y_{21n} = \frac{-2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$T_{22} = \frac{\left[1-Y_{11n}\left[\frac{Z_{01}^*}{Z_{01}}\right]\right]\left[1-Y_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]-Y_{12n}Y_{21n}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]}{2Y_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad Y_{22n} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}\right](T_{11}+T_{12})+(T_{21}+T_{22})}{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}$$

$$Y_{11n} = Y_{11}Z_{01} \qquad Y_{12n} = Y_{12}(Z_{01}Z_{02})^{1/2} \qquad Y_{21n} = Y_{21}(Z_{01}Z_{02})^{1/2} \qquad Y_{22n} = Y_{22}Z_{02}$$

$$T_{11} = \frac{(-h_{11n}-1)(1+h_{22n})+h_{12n}h_{21n}}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad h_{11n} = \frac{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$T_{12} = \frac{(h_{11n}+1)\left[1-h_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]+h_{12n}h_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad h_{12n} = \frac{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}(T_{11}T_{22}-T_{12}T_{21})}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$T_{21} = \frac{\left[\frac{Z_{01}^*}{Z_{01}}-h_{11n}\right](1+h_{22n})+h_{12n}h_{21n}}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad h_{21n} = \frac{-2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$T_{22} = \frac{\left[h_{11n}-\frac{Z_{01}^*}{Z_{01}}\right]\left[1-h_{22n}\left[\frac{Z_{02}^*}{Z_{02}}\right]\right]+h_{12n}h_{21n}\left[\frac{Z_{02}^*}{Z_{02}}\right]}{2h_{21n}\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]^{1/2}} \qquad h_{22n} = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}$$

$$h_{11n} = \frac{h_{11}}{Z_{01}} \qquad h_{12n} = h_{12}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2} \qquad h_{21n} = h_{21}\left[\frac{Z_{02}}{Z_{01}}\right]^{1/2} \qquad h_{22n} = h_{22}Z_{02}$$

$$T_{11} = \frac{A_n+B_n+C_n+D_n}{2} \qquad A_n = \frac{\left[\frac{Z_{01}^*}{Z_{01}}\right](T_{11}+T_{12})+(T_{21}+T_{22})}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$T_{12} = \frac{A_n\left[\frac{Z_{02}^*}{Z_{02}}\right]-B_n+C_n\left[\frac{Z_{02}^*}{Z_{02}}\right]-D_n}{2} \qquad B_n = \frac{T_{11}\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]-T_{12}\left[\frac{Z_{01}^*}{Z_{01}}\right]+T_{21}\left[\frac{Z_{02}^*}{Z_{02}}\right]-T_{22}}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$T_{21} = \frac{A_n+B_n-C_n\left[\frac{Z_{01}^*}{Z_{01}}\right]-D_n\left[\frac{Z_{01}^*}{Z_{01}}\right]}{2} \qquad C_n = \frac{T_{11}+T_{12}-T_{21}-T_{22}}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$T_{22} = \frac{A_n\left[\frac{Z_{02}^*}{Z_{02}}\right]-B_n-C_n\left[\frac{Z_{01}^*Z_{02}^*}{Z_{01}Z_{02}}\right]+D_n\left[\frac{Z_{01}^*}{Z_{01}}\right]}{2} \qquad D_n = \frac{\left[\frac{Z_{02}^*}{Z_{02}}\right](T_{11}-T_{21})-(T_{12}-T_{22})}{2\left[\frac{R_{01}R_{02}}{Z_{01}Z_{02}}\right]}$$

$$A_n = \frac{AZ_{02}}{(R_{01}R_{02})^{1/2}} \qquad B_n = \frac{B}{(R_{01}R_{02})^{1/2}} \qquad C_n = \frac{CZ_{01}Z_{02}}{(R_{01}R_{02})^{1/2}} \qquad D_n = \frac{DZ_{01}}{(R_{01}R_{02})^{1/2}}$$

TABLE V
EQUATIONS SHOWING THE CONVERSIONS BETWEEN Z, Y, h, AND $ABCD$ PARAMETERS

$Z_{11} = \frac{Y_{22}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{11} = \frac{h_{11}h_{22}-h_{12}h_{21}}{h_{22}}$	$h_{11} = \frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{22}}$
$Z_{12} = \frac{-Y_{12}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{12} = \frac{-Z_{12}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{12} = \frac{h_{12}}{h_{22}}$	$h_{12} = \frac{Z_{12}}{Z_{22}}$
$Z_{21} = \frac{-Y_{21}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{21} = \frac{-Z_{21}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{21} = \frac{-h_{21}}{h_{22}}$	$h_{21} = \frac{-Z_{21}}{Z_{22}}$
$Z_{22} = \frac{Y_{11}}{Y_{11}Y_{22}-Y_{12}Y_{21}}$	$Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22}-Z_{12}Z_{21}}$	$Z_{22} = \frac{1}{h_{22}}$	$h_{22} = \frac{1}{Z_{22}}$
$Z_{11} = \frac{A}{C}$	$A = \frac{Z_{11}}{Z_{21}}$	$Y_{11} = \frac{1}{h_{11}}$	$h_{11} = \frac{1}{Y_{11}}$
$Z_{12} = \frac{AD-BC}{C}$	$B = \frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}}$	$Y_{12} = \frac{-h_{12}}{h_{11}}$	$h_{12} = \frac{-Y_{12}}{Y_{11}}$
$Z_{21} = \frac{1}{C}$	$C = \frac{1}{Z_{21}}$	$Y_{21} = \frac{h_{21}}{h_{11}}$	$h_{21} = \frac{Y_{21}}{Y_{11}}$
$Z_{22} = \frac{D}{C}$	$D = \frac{Z_{22}}{Z_{21}}$	$Y_{22} = \frac{h_{11}h_{22}-h_{12}h_{21}}{h_{11}}$	$h_{22} = \frac{Y_{11}Y_{22}-Y_{12}Y_{21}}{Y_{11}}$
$Y_{11} = \frac{D}{B}$	$A = \frac{-Y_{22}}{Y_{21}}$	$h_{11} = \frac{B}{D}$	$A = \frac{h_{12}h_{21}-h_{11}h_{22}}{h_{21}}$
$Y_{12} = \frac{BC-AD}{B}$	$B = \frac{-1}{Y_{21}}$	$h_{12} = \frac{AD-BC}{D}$	$B = \frac{-h_{11}}{h_{21}}$
$Y_{21} = \frac{-1}{B}$	$C = \frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}}$	$h_{21} = \frac{-1}{D}$	$C = \frac{-h_{22}}{h_{21}}$
$Y_{22} = \frac{A}{B}$	$D = \frac{-Y_{11}}{Y_{21}}$	$h_{22} = \frac{C}{D}$	$D = \frac{-1}{h_{21}}$

between T parameters and Z, Y, h, and $ABCD$ parameters. Tables III and IV provide the conversions from S and T parameters to the normalized Z, Y, h, and $ABCD$ parameters, respectively. From Tables III and IV, it is easy to see that if Z_{01} and Z_{02} are real, the conversions become those shown in many of the references cited, e.g., [2], [4], [7], [8], [11], [12], [14], [15]. Finally, Table V shows the conversions between Z, Y, h, and $ABCD$ parameters while Table VI shows the conversions between S and T parameters. These are included to make the table of conversions in this paper complete.

IV. VERIFICATION

Using PSPICE, a SPICE based circuit analysis program, a lumped element model of an NE32000 HEMT was analyzed. The netlist was taken from the NEC databook and is shown below:

```
gl 5 6 3 4 0.045
lg 1 2 0.1nh
rg 2 3 2
cgs 3 4 0.2pf
cgd 3 5 0.016pf
cdg 5 4 6.7ff
ri 4 6 4
rs 6 7 3.5
ls 7 10 0.03nh
rds 5 6 200
cds 5 6 7.2ff
rd 5 8 4
ld 8 9 0.09nh.
```

By properly configuring a source at first port 1 then port 2, and opening and shorting out the other port, PSPICE will provide the complex voltages and currents required to calculate the Z, Y, h, and $ABCD$ parameters. Tables VII and VIII show the voltages and currents from PSPICE under the conditions listed in those tables. The Z, Y, h, and $ABCD$ parameters are calculated from these using (1)–(4) and are shown in Table IX.

The NE32000 lumped element model was also analyzed using Super Compact. For no particular reason, I chose to

TABLE VI
EQUATIONS SHOWING THE CONVERSIONS BETWEEN S AND T PARAMETERS

$S_{11} = \frac{T_{21}}{T_{11}}$	$T_{11} = \frac{1}{S_{21}}$
$S_{12} = \frac{T_{11}T_{22}-T_{12}T_{21}}{T_{11}}$	$T_{12} = \frac{-S_{22}}{S_{21}}$
$S_{21} = \frac{1}{T_{11}}$	$T_{21} = \frac{S_{11}}{S_{21}}$
$S_{22} = \frac{-T_{12}}{T_{11}}$	$T_{22} = \frac{S_{12}S_{21}-S_{11}S_{22}}{S_{21}}$

calculate the S parameters for the NE32000 in a system with a source impedance, Z_{01}, equal to $70+j\ 30$ and load impedance, Z_{02}, equal to $25-j\ 35$ at the single frequency of 10 GHz. The results of the Super Compact analysis are shown in Table X.

If a person uses the Z, Y, h, or $ABCD$ parameters of Table IX, in the equations of Table I, with $Z_{01} = 70+j\ 30$ and $Z_{02} = 25-j\ 35$, they will find that the calculated S parameters agree with those from Super Compact. In a like fashion, using the S parameters of Super Compact in the other equations in Table I will result in Z, Y, h, and $ABCD$ parameters shown in Table IX.

V. CONCLUSION

This paper developed the equations for converting between the various common 2-port parameters, Z, Y, h, $ABCD$, S, and T. The equations are derived from the definitions of the various 2-port parameters, the definition of a_j and b_j, and basic transmission line theory. As a result, the equations are completely general and are valid for complex and unique source and load impedances.

The validity of these results is shown by first calculating S parameters from Z, Y, h, and $ABCD$ parameters for an NE32000 HEMT in a system with $Z_S = 70 + j\ 30$ and $Z_L = 25 - j\ 35$. These results agreed with the S parameters produced by Super Compact. Also, beginning with the S parameters from Super Compact, the Z, Y, h, and $ABCD$ parameters are calculated using the equations developed. The results are the same as those calculated from the voltages and currents produced by PSPICE.

TABLE VII
VOLTAGES AND CURRENTS FOR THE NE32000 HEMT AT 10 GHz WITH THE SOURCE AT PORT 1. THE VOLTAGES AND CURRENTS ARE DEFINED IN FIG. 1

$V_1 = 1 + j\,0$			
$I_2 = 0$ (Port 2 Open Circuited)		$V_2 = 0$ (Port 2 Short Circuited)	
I_1	V_2	I_1	I_2
$8.844E-03 + j\,2.371E-02$	$-8.181E+00 + j\,5.615E+00$	$2.010E-03 + j\,1.292E-02$	$4.018E-02 - j\,1.071E-02$

TABLE VIII
VOLTAGES AND CURRENTS FOR THE NE32000 HEMT AT 10 GHz WITH THE SOURCE AT PORT 2. THE VOLTAGES AND CURRENTS ARE DEFINED IN FIG. 1

$V_2 = 1 + j\,0$			
$I_1 = 0$ (Port 1 Open Circuited)		$V_1 = 0$ (Port 1 Short Circuited)	
I_2	V_1	I_2	I_1
$8.032E-03 + j\,1.119E-03$	$9.661E-02 + j\,1.869E-02$	$3.949E-03 + j\,1.402E-03$	$4.741E-05 - j\,1.286E-03$

TABLE IX
Z, Y, h, AND $ABCD$ PARAMETERS FOR THE NE3200 HEMT AT 10 GHz. THESE PARAMETERS WERE CALCULATED FROM THE VOLTAGES AND CURRENTS IN TABLES VII AND VIII USING (1)–(4)

	11	12	21	22
Z	$1.380E+01 - j\,3.702E+01$	$1.212E+01 + j\,6.395E-01$	$9.518E+01 + j\,3.803E+02$	$1.221E+02 - j\,1.701E+01$
Y	$2.010E-03 + j\,1.292E-02$	$4.741E-05 - j\,1.286E-03$	$4.018E-02 - j\,1.071E-02$	$3.949E-03 + j\,1.402E-03$
h	$1.176E+01 - j\,7.557E+01$	$9.661E-02 + j\,1.869E-02$	$-3.370E-01 - j\,3.162E+00$	$8.032E-03 + j\,1.119E-03$
	A	B	C	D
$ABCD$	$-8.309E-02 - j\,5.703E-02$	$-2.324E+01 - j\,6.194E+00$	$6.173E-04 - j\,2.474E-03$	$3.332E-02 - j\,3.127E-01$

TABLE X
SUPER COMPACT RESULTS FOR THE NE32000 HEMT

		$Z_S = 70 + j\,30$					$Z_L = 25 - j\,35$		
		MICROWAVE HARMONICA PC V1.06 File: ne320-l.ckt 25-FEB-92 21:42:46							
Freq GHz	MS11 mag NE320L	PS11 deg NE320L	MS21 mag NE320L	PS21 deg NE320L	MS12 mag NE320L	PS12 deg NE320L	MS22 mag NE320L	PS22 deg NE320L	MS21 dB NE320L
10.000	0.665	−121.4	2.194	118.3	0.068	45.3	0.796	−12.4	6.82

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